Revisiting Neighborhood Inverse Consistency on Binary CSPs

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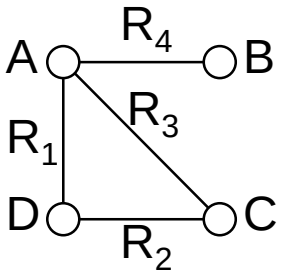
Outline

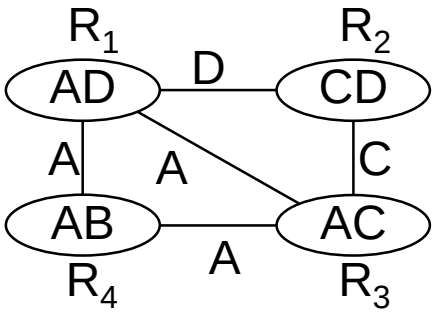
- Introduction: Relational NIC
- Structure of the dual graph of a binary CSP affects RNIC
- RNIC versus NIC, sCDC on binary CSPs
- Experimental results
- Conclusion

Constraint Satisfaction Problem

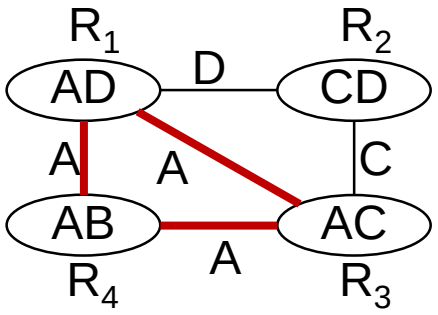
- Graphical Representation
  - Constraint graph
  - Dual graph
  - Minimal dual graph



Constraint graph



Dual graph

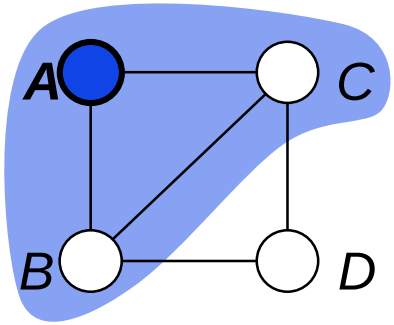


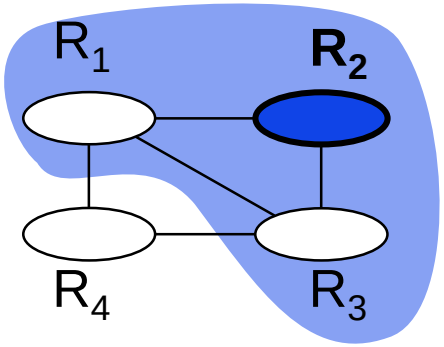
Minimal dual graph

[Janssen+,1989]

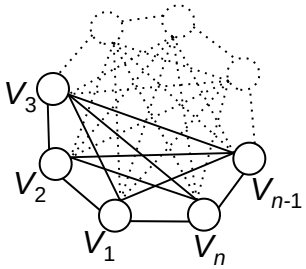
Neighborhood Inverse Consistency

- Relational NIC [Woodward+ AAAI 2011]
  - Reformulation of NIC [Freuder & Elfe, AAAI 1996]
  - Defined for dual graph
  - Every tuple can be extended to a solution in its relation's neighborhood
  - Algorithm operates on dual graph & filter relations (not domains!)
  - Initially designed for non-binary CSPs
- How about RNIC on binary CSPs?
  - Impact of the structure of the dual graph
  - RNIC versus other consistency properties

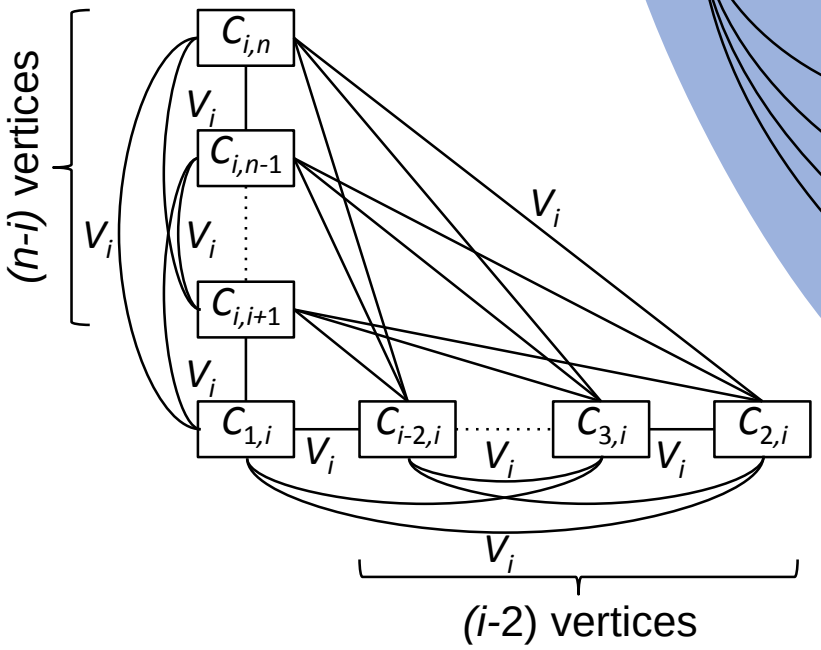


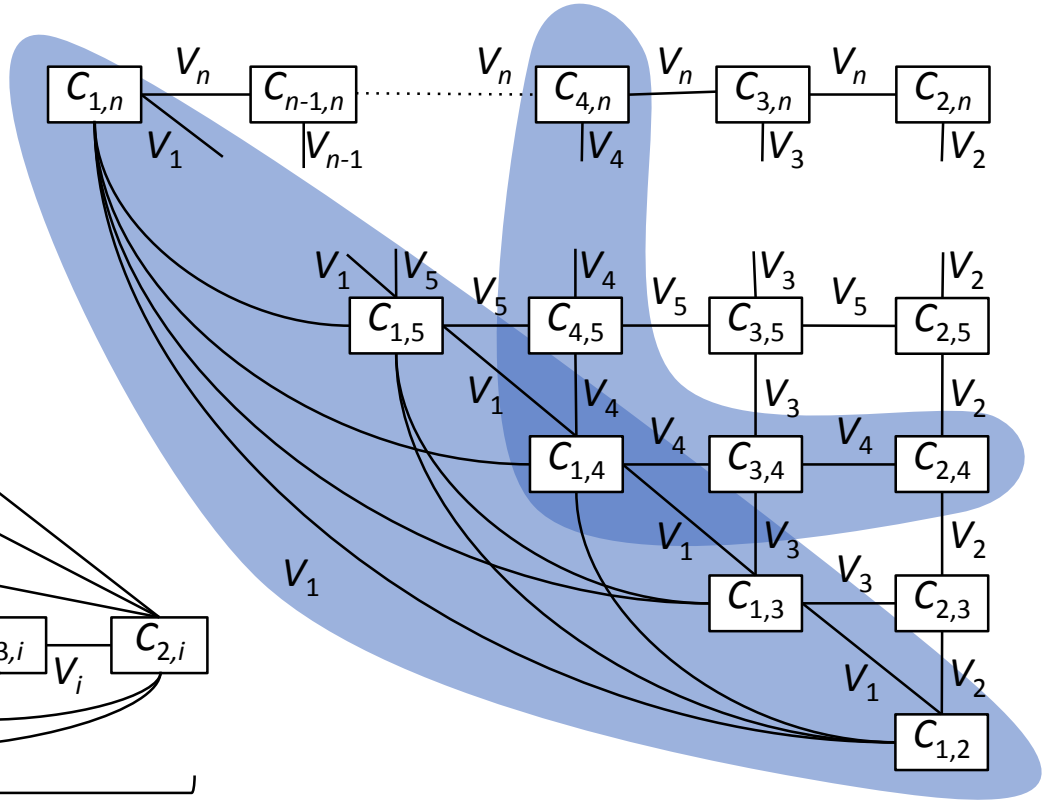


Complete Constraint Graph

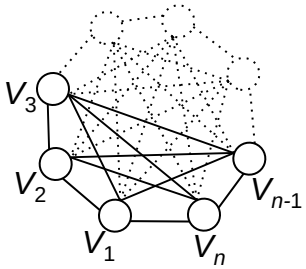


Dual Graph: Triangle shaped grids

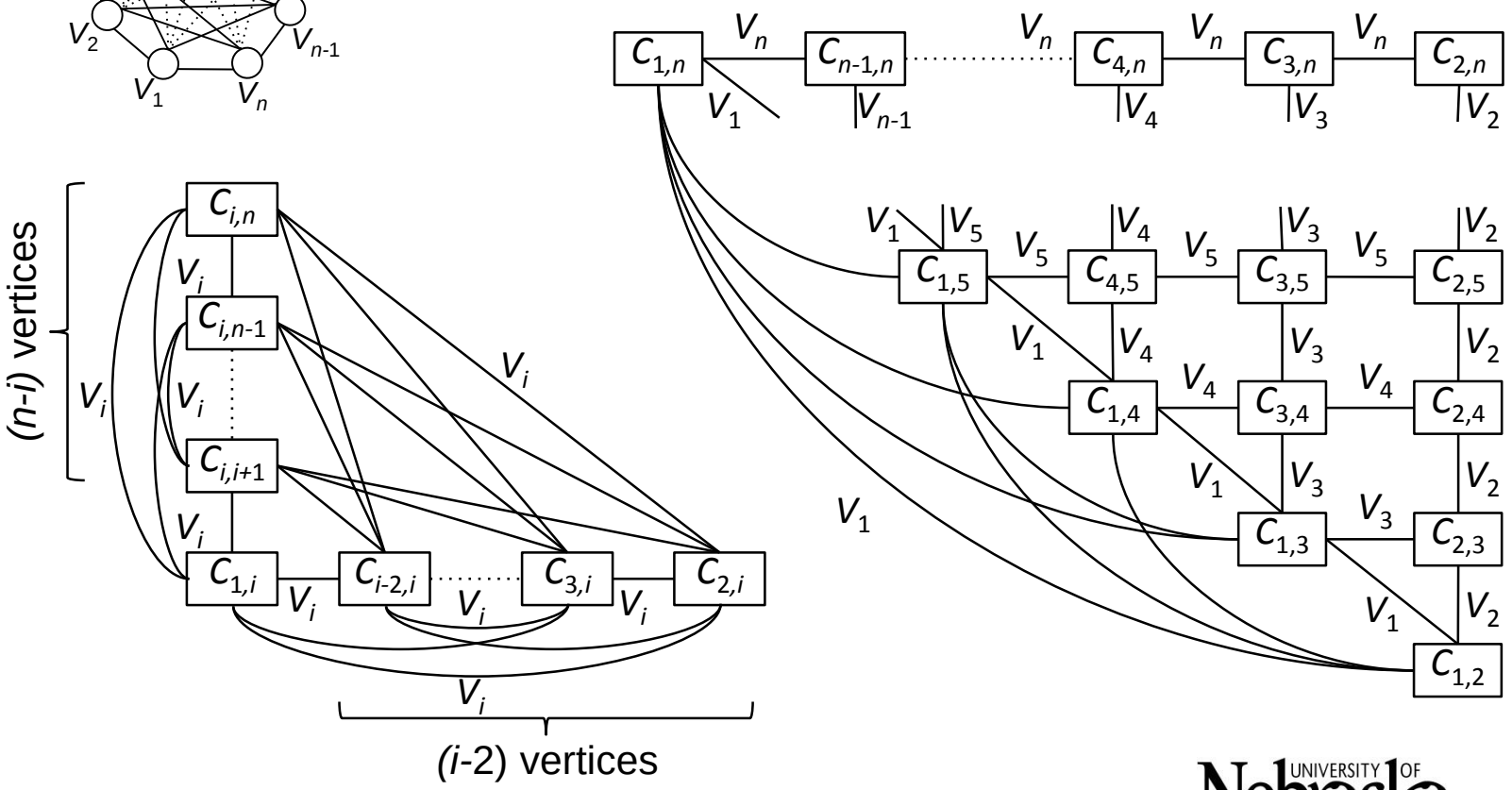




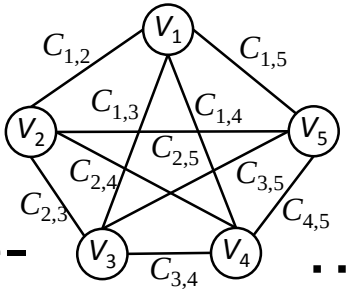
Minimal Dual Graph



Dual Graph: Triangle shaped grids



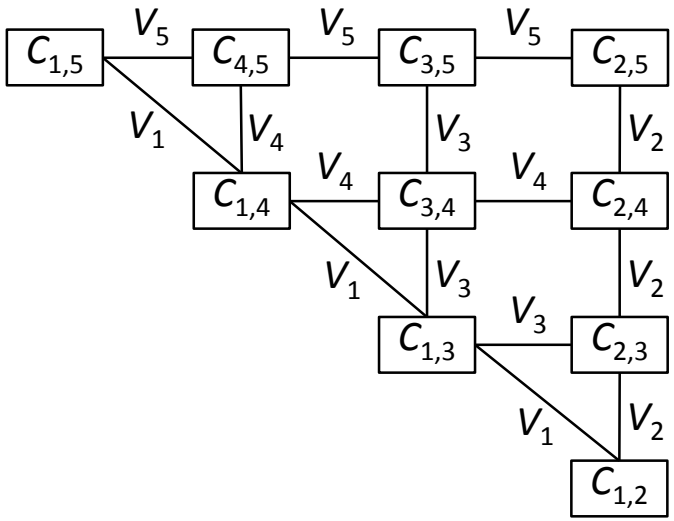
Minimal Dual Graph

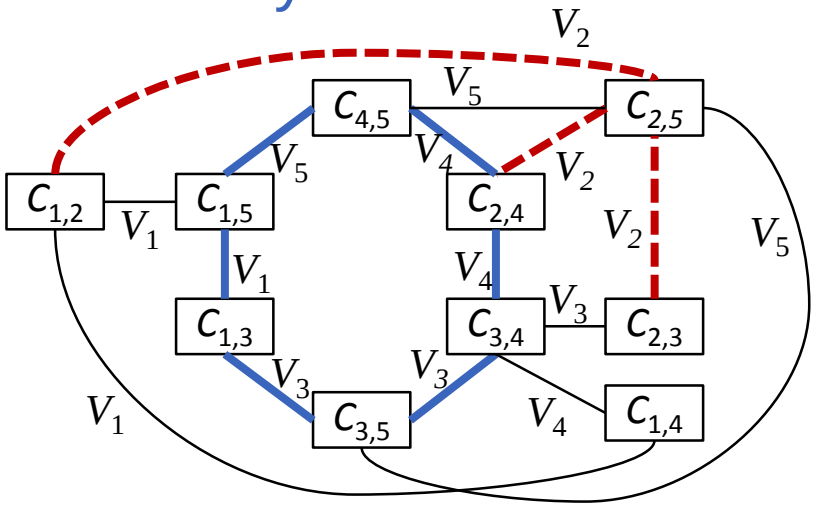


… can be a triangle-shaped grid (planar)

… but does not have to be

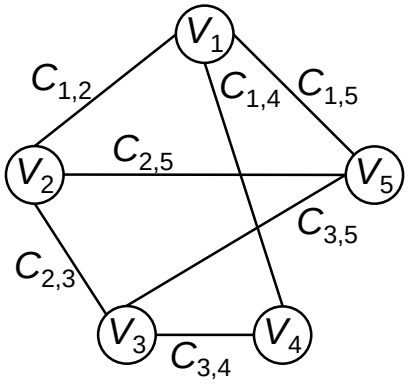
- Star on V_2
- Cycle of size 6

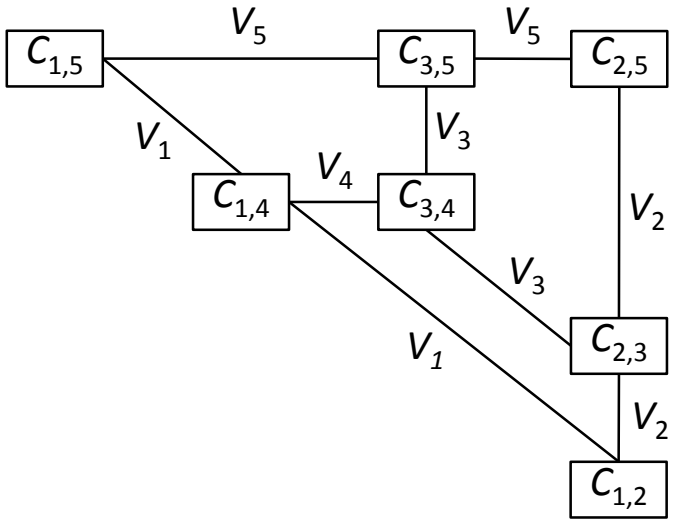




Non-Complete Constraint Graph

- Can still be a triangle-shaped grid
  - Have a chain of vertices
  - of length ≤ *n*-1

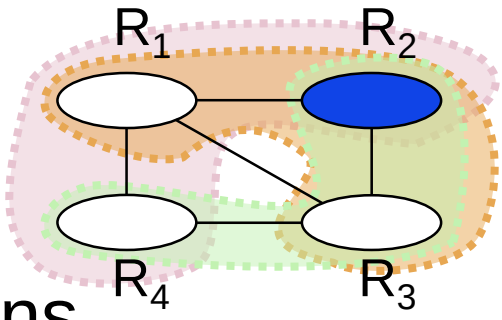




Impact on RNIC

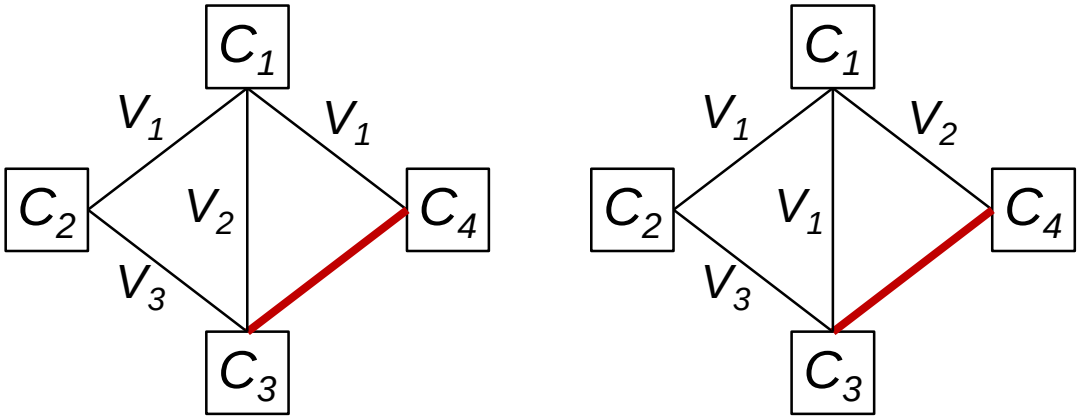
On a binary CSP, RNIC enforced on the minimal dual graph (wRNIC) is never strictly stronger than R(,3)C.*

- R(*,*m*)C ensures that subproblem induced on the dual CSP by every connected combination of *m* relations is minimal [Karakashian+, AAAI 2010]



wRNIC on Binary CSPs

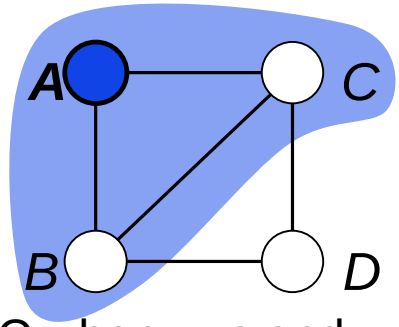
- wRNIC can never consider more than 3 relations



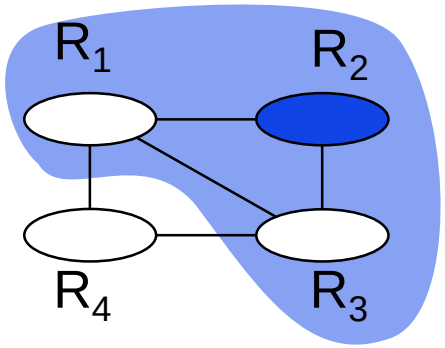
- In either case, it is not possible to have an edge between C_3 & C_4 (a common variable to C_3 & C_4) while keeping C_3 as a binary constraint

NIC, sCDC, and RNIC not comparable

- NIC Property [Freuder & Elfe, AAAI 1996]
  - ↪ Every value can be extended to a solution in its variable's neighborhood



- sCDC Property [Lecoutre+, JAIR 2011]
  - ↪ An instantiation {(*x,a*),(*y,b*)} is DC iff (*y,b*) holds in SAC when *x*=*a* and (*x,a*) holds in SAC when *y*=*b* and (*x,y*) in scope of some constraint. Further, the problem is also AC.
- RNIC Property [Woodward+, AAAI 2011]
  - ↪ Every tuple can be extended to a solution in its relation's neighborhood
  - ↪ wRNIC, triRNIC, wtriRNIC enforce RNIC on a minimal, triangulated, and minimal triangulated dual graph, respectively
  - ↪ **selRNIC** automatically selects the RNIC variant based on the density of the dual graph



Experimental Results (CPU Time)

Benchmark	# inst.	AC3.1	sCDC1	NIC	selRNIC
		CPU Time (msec)			
		NIC Quickest			
bqwh-16-106	100/100	3,505	3,860	**1,470**	3,608
bqwh-18-141	100/100	68,629	82,772	**38,877**	77,981
coloring-sgb-queen	12/50	680,140	(+3) -	**(+9) 57,545**	634,029
coloring-sgb-games	3/4	41,317	33,307	**(+1) 860**	41,747
rand-2-23	10/10	1,467,246	1,460,089	**987,312**	1,171,444
rand-2-24	3/10	**567,620**	677,253	**(+7)** 3,456,437	677,883
		sCDC Quickest			
driver	2/7	(+5) 70,990	**(+5) 17,070**	358,790	(+4) 185,220
ehi-85	87/100	(+13) 27,304	**(+13) 573**	513,459	(+13) 75,847
ehi-90	89/100	(+11) 34,687	**(+11) 605**	713,045	(+11) 90,891
frb35-17	10/10	41,249	**38,927**	179,763	73,119
		RNIC Quickest			
composed-25-1-25	10/10	226	335	1,457	**114**
composed-25-1-2	10/10	223	283	1,450	**88**
composed-25-1-40	9/10	(+1) 288	(+1) 357	120,544	**(+1) 137**
composed-25-1-80	10/10	223	417	(+1) -	**190**
composed-75-1-25	10/10	2,701	1,444	363,785	**305**
composed-75-1-2	10/10	2,349	1,733	48,249	**292**
composed-75-1-40	7/10	(+1) 1,924	(+3) 1,647	631,040	**(+3) 286**
composed-75-1-80	10/10	1,484	1,473	(+1) -	**397**

Experimental Results (BT-free, #NV)

Benchmark	# inst.	AC3.1	sCDC1	NIC	selRNIC	AC3.1	sCDC1	NIC	selRNIC
		BT-Free				#NV			
		NIC Quickest							
bqwh-16-106	100/100	0	3	**8**	5	1,807	1,881	**739**	1,310
bqwh-18-141	100/100	0	0	**1**	0	25,283	25,998	**12,490**	22,518
coloring-sgb-queen	12/50	1	-	**16**	1	91,853	-	**15,798**	91,853
coloring-sgb-games	3/4	1	1	**4**	1	14,368	14,368	**40**	14,368
rand-2-23	10/10	0	0	**10**	0	471,111	471,111	**12**	471,111
rand-2-24	3/10	0	0	**10**	0	222,085	222,085	**24**	222,085
		sCDC Quickest							
driver	2/7	1	**2**	1	1	3,893	**409**	3,763	3,763
ehi-85	87/100	0	**100**	87	**100**	1,425	**0**	**0**	**0**
ehi-90	89/100	0	**100**	89	**100**	1,298	**0**	**0**	**0**
frb35-17	10/10	0	0	0	0	24,491	24,491	24,491	**24,346**
		RNIC Quickest							
composed-25-1-25	10/10	0	**10**	**10**	**10**	153	**0**	**0**	**0**
composed-25-1-2	10/10	0	**10**	**10**	**10**	162	**0**	**0**	**0**
composed-25-1-40	9/10	0	**10**	9	**10**	172	**0**	**0**	**0**
composed-25-1-80	10/10	0	**10**	-	**10**	112	**0**	-	**0**
composed-75-1-25	10/10	0	**10**	**10**	**10**	345	**0**	**0**	**0**
composed-75-1-2	10/10	0	**10**	**10**	**10**	346	**0**	**0**	**0**
composed-75-1-40	7/10	0	**10**	7	**10**	335	**0**	**0**	**0**
composed-75-1-80	10/10	0	**10**	-	**10**	199	**0**	-	**0**

Conclusions

- Contributions
  - Apply RNIC to binary CSPs
  - Structure of dual graph & impact of RNIC
  - NIC, sCDC, and RNIC are incomparable
  - Empirically shown benefits of higher-level consistencies
- Future work
  - Study impact of the structure of the dual graph on (future) relational consistency properties
  - ‘Predict’ appropriate consistency property using information about the problem and its structure